Blade Design for Swirling Flow Generator

Romeo SUSAN-RESIGA[1], Sebastian MUNTEAN[2] Alin BOSIOC[1],

[(1)] National Center for Engineering of Systems with Complex Fluids, “Politehnica” University of Timişoara, Bd. Mihai Viteazu, 1, 300222, Timişoara, Romania

[(2)] Center for Advanced Research in Engineering Sciences, Romanian Academy – Timişoara Branch, Bd. Mihai Viteazu, 24, 300223, Timişoara, Romania

Abstract

The paper briefly recall the classical theory for quasi-3D turbomachinery design and analysis, then presents an application for a swirl generator blades. Both guide vanes and free-runner blades are designed using an inverse quasi-three-dimensional method.

Axisymmetric Turbomachinery Swirling Flow

The preliminary analysis and design of turbomachinery flow can be performed within the inviscid fluid assumption, since losses can be considered negligible at the design operating point. Of course, maximizing the machine efficiency requires the evaluation of viscous losses, but the first step is to get a preliminary design within the loss-free framework.

For the absolute steady flow (stay vanes, guide vanes, bladeless regions) the continuity and momentum equations are

$$\begin{aligned} &\nabla \cdot \boldsymbol{V} = 0 && \text{continuity equation,} \\ &(\nabla \times \boldsymbol{V}) \times \boldsymbol{V} = -\nabla E && \text{momentum equation,} \quad . \\ &E \equiv \frac{p}{\rho} + \frac{V^2}{2} && \text{specific energy per unit mass.} \end{aligned} \tag{1}$$

Obviously, the momentum equation gives $\boldsymbol{V} \cdot \nabla E = 0$, i.e. the specific mechanical energy remains constant along a streamline for the absolute velocity field.

For the runner bladed region it is convenient to consider the relative flow equations, where the relative velocity is $\boldsymbol{W} = \boldsymbol{V} - \boldsymbol{\Omega} \times \boldsymbol{r}$ and the absolute specific energy E is replaced by the relative specific energy $E_R = E - \Omega(rV_\theta)$. The corresponding steady relative flow equations are

$$\begin{aligned} &\nabla \cdot \boldsymbol{W} = 0 && \text{continuity for relative flow,} \\ &(\nabla \times \boldsymbol{V}) \times \boldsymbol{W} = -\nabla E_R && \text{momentum for relative flow,} . \\ &E_R \equiv \frac{p}{\rho} + \frac{W^2}{2} - \frac{(\Omega r)^2}{2} && \text{relative specific energy.} \end{aligned} \tag{2}$$

From the momentum equation for relative flow we have $\boldsymbol{W} \cdot \nabla E_R = 0$, i.e. the relative specific energy E_R remains constant along relative flow streamlines.

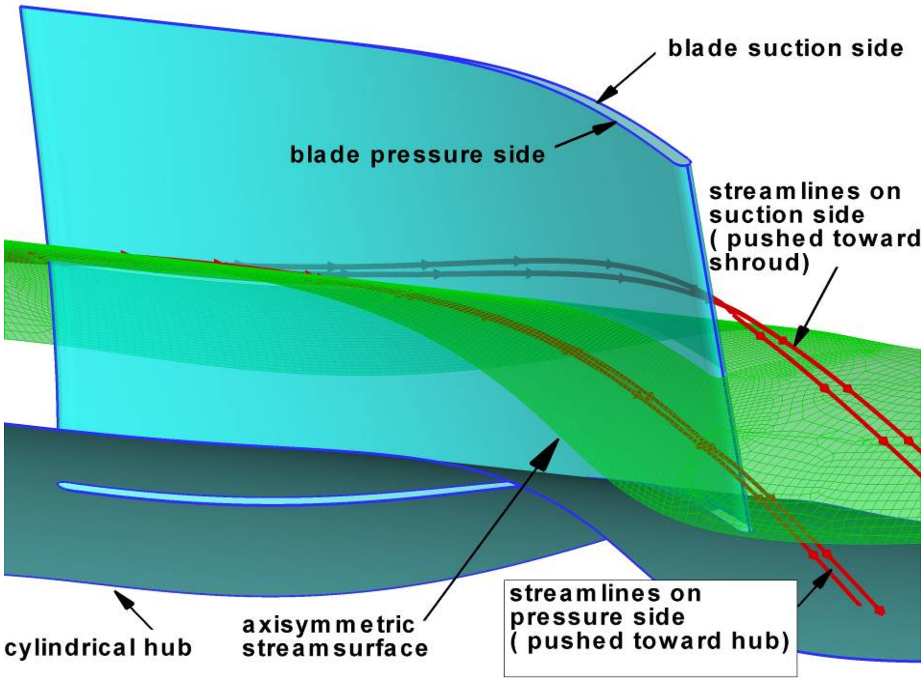

Fig. 1: Three-dimensional effects of blade loading and the axisymmetric flow assumption.

For the three-dimensional flow in the bladed regions, the blade produces a pressure difference between suction and pressure sides, which in turn produces a circumferential pressure gradient that deflects the flow. **Fig. 1** shows that the flow inside the interblade channel is essentially three-dimensional since the streamlines originating on a circle (normal to the machine axis) do not remain on an axisymmetric surface. Instead, the streamlines close to the blade pressure side are pushed radially towards the hub, while the streamlines in the neighborhood of blade suction side are deflected toward the shroud. It is obvious from **Fig. 1** that at the blade trailing edge there is a significant radial departure between streamlines originating at the same radius upstream the blade. However, a simplified axisymmetric model for the bub-to-shroud turbomachinery flow considers that the streamsurfaces retain axial symmetry within the blade regions as well.

The simplified axisymmetric computation retains only the average inter-blade pressure since no circumferential gradient is allowed. As a result, we need an artificial quantity, the *blade body force* $\boldsymbol{B}$, to account for the blade-flow interaction. This body force introduced in the axisymmetric flow model should turn the flow equivalently with the actual blades. The axisymmmetric absolute flow equations (1) become

$$\begin{aligned}
&\nabla\cdot\boldsymbol{V}=0 && \text{continuity equation,}\\
&(\nabla\times\boldsymbol{V})\times\boldsymbol{V}=-\nabla E+\boldsymbol{B} && \text{momentum equation,}\\
&E\equiv\frac{p}{\rho}+\frac{V^2}{2} && \text{specific energy per unit mass in absolute flow}
\end{aligned}\quad . \tag{3}$$

while the corresponding axisymmetric relative flow equations are

$$\begin{aligned}
&\nabla\cdot\boldsymbol{W}=0 && \text{continuity,}\\
&(\nabla\times\boldsymbol{V})\times\boldsymbol{W}=-\nabla E_R+\boldsymbol{B} && \text{momentum,}\\
&E_R\equiv\frac{p}{\rho}+\frac{W^2}{2}-\frac{(\Omega r)^2}{2} && \text{specific energy per unit mass in relative flow}
\end{aligned}\quad . \tag{4}$$

Within the bladed regions, the flow is considered to take place on an absolute or relative streamsurface defined as

$$\alpha(z,r,\theta)\equiv\theta-f(z,r)=\text{constant}\,, \tag{5}$$

where $f(z,r)$ defines the blade wrap angle, within the concept of infinite number of blades. Conceptually, the constant α surface corresponds to the S_2-surface concept proposed by Wu [6]. For example, a surface described as in Eq.(5) may be seen as corresponding to the thin blade from **Fig. 1**, in the limit of zero thickness. With the unit normal vector to the streamsurface,

$$\boldsymbol{n}\equiv\frac{\nabla\alpha}{|\nabla\alpha|}=\frac{\boldsymbol{e}_\theta-\frac{r\partial f}{\partial r}\boldsymbol{e}_r-\frac{r\partial f}{\partial z}\boldsymbol{e}_z}{\sqrt{1+\left(\frac{r\partial f}{\partial r}\right)^2+\left(\frac{r\partial f}{\partial z}\right)^2}}\,, \tag{6}$$

the flow tangency condition requires

$$\begin{aligned}
&\boldsymbol{n}\cdot\boldsymbol{V}=0 && \text{absolute flow}\\
&\boldsymbol{n}\cdot\boldsymbol{W}=0 && \text{relative flow}
\end{aligned}\quad . \tag{7}$$

The blade body force $\boldsymbol{B}$ is normal to either the absolute or relative velocity vector (i.e. along the normal direction to the absolute/relative streamsurface),

$$\begin{aligned}
&\boldsymbol{B}\cdot\boldsymbol{V}=0 && \text{absolute flow}\\
&\boldsymbol{B}\cdot\boldsymbol{W}=0 && \text{relative flow}
\end{aligned}\quad , \tag{8}$$

and it can be written as,

$$\boldsymbol{B}=B(z,r)\,\nabla\alpha,\ \text{with}\ B=\boldsymbol{V}\cdot\nabla(rV_\theta). \tag{9}$$

In Eq.(9) the function $B(z,r)$ is related to the change in the fluid's angular momentum imparted by the blade rows. One can easily check from Eq.(9) that the blade body force is measured in $\left[m/s^2\right]$, as it should in absolute/relative momentum equations. Since the constant α surface may be seen as an approximation for a thin blade surface, $\nabla\alpha$ is a vector normal to the blade surface. The blade body force $\boldsymbol{B}$ vanishes outside the blade region since rV_θ does not change anymore along the absolute streamlines. The definition (9) is consistent with the Euler equation of turbomachinery. The torque generated by the circumferential component of the blade body force, $B_\theta = \left[\boldsymbol{V}\cdot\nabla\left(rV_\theta\right)\right]/r$ is

$$T = \int_{Vol} r\rho B_\theta \,\mathrm{d}Vol = \int_{Vol} \left[\rho\boldsymbol{V}\cdot\nabla\left(rV_\theta\right)\right]\mathrm{d}Vol . \tag{10}$$

Note that thanks to the continuity equation, the term in the square brackets can be written as $\nabla\cdot\left[\rho\mathbf{V}\left(rV_\theta\right)\right]$. The power transferred by the runner, rotating with the angular speed Ω, from/to the fluid is

$$\begin{aligned} P = T\Omega &= \int_{Vol} \nabla\cdot\left[\rho\boldsymbol{V}\left(rV_\theta\right)\right]\mathrm{d}Vol = \int_S \left(\Omega r V_\theta\right)\rho\boldsymbol{V}\cdot\boldsymbol{n}\,dS \\ &= \int_{S_{IN}} \left(UV_\theta\right)\rho\boldsymbol{V}\cdot\boldsymbol{n}_{IN}\,dS + \int_{S_{OUT}} \left(UV_\theta\right)\rho\boldsymbol{V}\cdot\boldsymbol{n}_{OUT}\,dS \end{aligned} . \tag{11}$$

Eq.(11) is the fundamental turbomachinery equation. We have denoted the transport velocity $U = \Omega r$. After using the Gauss theorem to transform the volume integral into an integral over the domain boundary, we have taken into account that $\boldsymbol{V}\cdot\boldsymbol{n} = 0$ on the hub and shroud walls. Obviously, on the inlet section we have $\boldsymbol{V}\cdot\boldsymbol{n}_{IN} < 0$, while on the outlet section we have $\boldsymbol{V}\cdot\boldsymbol{n}_{OUT} > 0$. If mass weighted average values for $\overline{UV_\theta}$ are considered on the inlet/outlet section, one can write the fundamental turbomachinery equation in the simple form $P = \dot{m}\left[\overline{\left(UV_\theta\right)}_{OUT} - \overline{\left(UV_\theta\right)}_{IN}\right]$, with $\dot{m}$ the mass flow rate through the turbomachine.

We can now replace the three projections of the momentum equations by:

i) Flow tangency condition on the absolute or relative streamsurface, (7);

ii) Projection on the streamsurface, along the velocity vector, i.e. $\boldsymbol{V}\cdot\nabla E = 0$ for absolute flow and $\boldsymbol{W}\cdot\nabla E_R = 0$ for relative flow;

iii) Projection on the streamsurface, normal to the velocity vector. This projection leads to the main scalar equation for steady absolute or relative turbomachinery axisymmetric flow.

The projection of the momentum equation ii) shows that E_R does not change along a streamline in relative flow. Since by definition we have $E_R = E - UV_\theta$, the fundamental turbomachinery equation can be rewritten as $P = \dot{m}\left[\overline{E_{OUT}} - \overline{E_{IN}}\right]$, thus showing that, if no viscous losses are accounted for, the mechanical power transferred from/to the runner corresponds to the increase/decrease of the fluid specific energy E.

Let us focus now on iii), and follow the derivation of the main scalar equation for the relative flow. The direction on the streamsurface, normal to the velocity vector is given by the following unit vector

$$\boldsymbol{N} \equiv \frac{\boldsymbol{W}}{W} \times \boldsymbol{n}, \tag{12}$$

shown in **Fig. 2**.

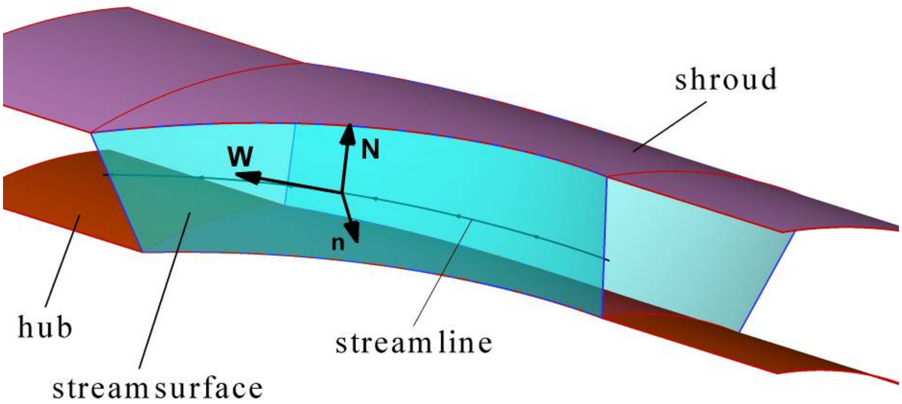

Fig. 2: Unit vectors on a streamsurface, as defined in Eqs. (6) and (12).

After a straightforward computation, and using the flow tangency condition (7),

$$\left(\frac{\boldsymbol{W}}{W} \times \boldsymbol{n}\right) \cdot \left[-\boldsymbol{W} \times (\nabla \times \boldsymbol{V})\right] = \left\{\left[-\boldsymbol{W} \times (\nabla \times \boldsymbol{V})\right] \times \frac{\boldsymbol{W}}{W}\right\} \cdot \boldsymbol{n}$$
$$= \frac{\boldsymbol{n}}{W} \cdot \left\{\boldsymbol{W} \times \left[\boldsymbol{W} \times (\nabla \times \boldsymbol{V})\right]\right\} = \frac{\boldsymbol{n}}{W} \cdot \left\{-W^2 (\nabla \times \boldsymbol{V}) + \left[\boldsymbol{W} \cdot (\nabla \times \boldsymbol{V})\right] \boldsymbol{W}\right\},$$
$$= -W \boldsymbol{n} \cdot (\nabla \times \boldsymbol{V})$$

we obtain the projection along $\boldsymbol{N}$ of the left-hand side of the momentum equation,

$$\boldsymbol{N} \cdot \left[(\nabla \times \boldsymbol{V}) \times \boldsymbol{W}\right] = -W \boldsymbol{n} \cdot (\nabla \times \boldsymbol{V}). \tag{13}$$

For the right-hand side of the momentum equation projection along $\boldsymbol{N}$ we have

$$(\boldsymbol{W} \times \boldsymbol{n}) \cdot \nabla E_R = -\frac{\partial E_R}{\partial r}\left(n_\theta W_z - n_z W_\theta\right) + \frac{\partial E_R}{\partial z}\left(n_\theta W_r - n_r W_\theta\right). \tag{14}$$

After recalling the vorticity vector in cylindrical coordinates,

$\nabla \times \boldsymbol{V} = -\frac{\partial V_\theta}{\partial z}\boldsymbol{e}_r + \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right)\boldsymbol{e}_\theta + \frac{1}{r}\frac{\partial (rV_\theta)}{\partial r}\boldsymbol{e}_z$, we use (13) and (14) to obtain the $\mathbf{N}-\text{projection}$ of the momentum equation, also called the *principal equation of motion*:

$$\begin{aligned} & n_\theta\left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right) + n_z\frac{1}{r}\frac{\partial (rV_\theta)}{\partial r} - n_r\frac{1}{r}\frac{\partial (rV_\theta)}{\partial z} \\ & = \frac{1}{W^2}\left[\frac{\partial E_R}{\partial z}\left(n_\theta W_r - n_r W_\theta\right) - \frac{\partial E_R}{\partial r}\left(n_\theta W_z - n_z W_\theta\right)\right] \end{aligned} . \quad (15)$$

From (6) we have the streamsurface normal orientation

$$\frac{n_z}{n_\theta} = -\frac{r\,\partial f}{\partial z} \text{ and } \frac{n_r}{n_\theta} = -\frac{r\,\partial f}{\partial r},$$

and the principal equation (15) can be rewritten as

$$\begin{aligned} & \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} = \frac{\partial f}{\partial z}\frac{\partial (rV_\theta)}{\partial r} - \frac{\partial f}{\partial r}\frac{\partial (rV_\theta)}{\partial z} \\ & + \frac{1}{W^2}\left[\frac{\partial E_R}{\partial z}\left(W_r + \frac{r\,\partial f}{\partial r}W_\theta\right) - \frac{\partial E_R}{\partial r}\left(W_z + \frac{r\,\partial f}{\partial z}W_\theta\right)\right] \end{aligned} . \quad (16)$$

Although currently used since '70s, Eq.(16) is still the basic mathematical model for hydraulic turbomachinery primary design, [4][5]. Bosman [2] has a word of caution concerning Eq.(16): where the streamsurface tangent plane subtends a large angle to the meridional plane, i.e. (n_r / n_θ) and (n_z / n_θ) become large, then Eq.(16) fails to converge numerically due to the amplification of the right-hand side by these factors. This situation is anticipated to be more prevalent in radial-flow machines.

For practical use it is convenient to re-write (16) using the Stokes' streamfunction Ψ for axisymmetric flows. Within the blade regions of a turbomachine the finite blade thickness is accounted for through the dimensionless *blade blockage coefficient* $b<1$. Within blade-less regions we obviously have $b=1$. From the circumferentially averaged continuity equation [3],

$$\frac{\partial (brV_z)}{\partial z} + \frac{\partial (brV_r)}{\partial r} = 0, \quad (17)$$

we can introduce the streamfunction as,

$$\frac{\partial \Psi}{\partial r} = rbV_z \text{ and } \frac{\partial \Psi}{\partial z} = -rbV_r \text{ for absolute flow, and} \quad (18)$$

$$\frac{\partial \Psi}{\partial r} = rbW_z \text{ and } \frac{\partial \Psi}{\partial z} = -rbW_r \text{ for relative flow.}$$

As a result, the principal equation for turbomachinery swirling flow Eq.(16) can be re-written as

$$\begin{aligned}&\frac{\partial}{\partial z}\left(\frac{1}{br}\frac{\partial \Psi}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{br}\frac{\partial \Psi}{\partial r}\right)=\frac{\partial f}{\partial r}\frac{\partial\left(rV_\theta\right)}{\partial z}-\frac{\partial f}{\partial z}\frac{\partial\left(rV_\theta\right)}{\partial r}\\&+\frac{1}{W^2}\left[\frac{\partial E_R}{\partial r}\left(W_z+\frac{r\partial f}{\partial z}W_\theta\right)-\frac{\partial E_R}{\partial z}\left(W_r+\frac{r\partial f}{\partial r}W_\theta\right)\right]\end{aligned} \tag{19}$$

In this form, one has to solve an elliptic partial differential equation for the streamfunction Ψ, with the corresponding source term in the right-hand side. Usually, the source term is treated explicitly within an iterative procedure. However, for loss-free flows the term

$$\frac{1}{W^2}\left[\frac{\partial E_R}{\partial r}\left(W_z+\frac{r\partial f}{\partial z}W_\theta\right)-\frac{\partial E_R}{\partial z}\left(W_r+\frac{r\partial f}{\partial r}W_\theta\right)\right],$$

can be treated implicitly. Since E_R depends in this case only on the streamfunction (being constant along the streamlines in the meridian half-plane), we have

$$\frac{\partial E_R}{\partial r}=\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\frac{\partial \Psi}{\partial r}=rbW_z\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}, \text{ and } -\frac{\partial E_R}{\partial z}=\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\left(-\frac{\partial \Psi}{\partial z}\right)=rbW_r\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}.$$

Note that when losses are accounted for, E_R (or E) does not remain constant along a relative (absolute) axisymmetric streamtube; it actually decreases downstream due to viscous dissipation.

From the flow tangency condition (7) we have

$$\boldsymbol{W}\cdot\nabla\alpha=\frac{W_\theta}{r}-W_z\frac{\partial f}{\partial z}-W_r\frac{\partial f}{\partial r}=0, \text{ or } W_\theta=W_z\frac{r\partial f}{\partial z}+W_r\frac{r\partial f}{\partial r}.$$

A straightforward computation gives,

$$\begin{aligned}&\frac{1}{W^2}\left[\frac{\partial E_R}{\partial r}\left(W_z+\frac{r\partial f}{\partial z}W_\theta\right)-\frac{\partial E_R}{\partial z}\left(W_r+\frac{r\partial f}{\partial r}W_\theta\right)\right]\\&=\frac{1}{W^2}\left[rbW_z\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\left(W_z+\frac{r\partial f}{\partial z}W_\theta\right)+rbW_r\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\left(W_r+\frac{r\partial f}{\partial r}W_\theta\right)\right]\\&=rb\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\frac{1}{W^2}\left[W_z^2+W_r^2+W_\theta\left(W_z\frac{r\partial f}{\partial z}+W_r\frac{r\partial f}{\partial r}\right)\right]\\&=rb\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\frac{W_z^2+W_r^2+W_\theta^2}{W^2}=rb\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}\end{aligned}.$$

As a result, the principal equation for loss-free axisymmetric turbomachinery swirling flows becomes

$$\frac{\partial}{\partial z}\left(\frac{1}{br}\frac{\partial \Psi}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{br}\frac{\partial \Psi}{\partial r}\right)-rb\frac{\mathrm{d}E_R}{\mathrm{d}\Psi}=\frac{\partial f}{\partial r}\frac{\partial\left(rV_\theta\right)}{\partial z}-\frac{\partial f}{\partial z}\frac{\partial\left(rV_\theta\right)}{\partial r} \quad (20)$$

For a given $\left(rV_\theta\right)$ distribution in the bladed region, Eq.(20) provides a relationship between the streamfunction Ψ and the relative flow streamsurface shape f. The term $\mathrm{d}E_R/\mathrm{d}\Psi$ is known as function of Ψ from the upstream conditions. The second relationship between Ψ and f is given by the flow tangency condition (7) rewritten as

$$\left(\frac{1}{rb}\frac{\partial \Psi}{\partial r}\right)\frac{\partial f}{\partial z}-\left(\frac{1}{rb}\frac{\partial \Psi}{\partial z}\right)\frac{\partial f}{\partial r}=\frac{V_\theta}{r}-\Omega\,. \quad (21)$$

The system of partial differential equations (20) and (21), with appropriate boundary conditions, allows the computation of both $\Psi(z,r)$ and $f(z,r)$ in the runner region. Borges [1] uses this mathematical model to design mixed-flow pumps. By assuming a uniform non-swirling inlet flow, the term $\mathrm{d}E_R/\mathrm{d}\Psi$ vanishes. Also, when thin blades are considered in a first approximation, the blade blockage coefficient is $b=1$. Borges discretizes the corresponding simplified equation (20) with a second-order accurate finite difference scheme, resulting in a nine-point stencil on a structured quadrilateral grid of the meridional domain. Equation (21) is a first order partial differential equation with characteristic lines coincident with the streamlines $\Psi=\text{constant}$ in the meridian half-plane. In order to integrate this differential equation, some initial data must be specified along a line roughly perpendicular to these characteristic lines and extending from hub to shroud. This initial data on f are the stacking condition of the blade. Borges [1] implements this condition by giving, as input, the values of the blade angular coordinate f along the impeller blade leading edge. Zangeneh [7] also uses the system of equations (20) and (21), but the blade camber surface is no longer approximated by the constant α streamsurface. In this case, the velocity field is decomposed into circumferentially averaged and periodic components, by using the Clebsh formulation of steady rotational flow. The blade shape is determined by imposing the inviscid slip condition (i.e. blade shape aligned with the local velocity vector), and it is slightly different from the constant α streamsurface.

Summary of Turbomachinery Swirling Flow Equations

Let us summarize now the mathematical model for turbomachinery swirling flow, derived within the following simplified assumptions:

- Incompressible and inviscid fluid; no hydraulic losses are accounted for

- Axi-symmetrical steady swirling flow; this assumption can be seen as considering an infinite number of zero thickness blades.

In order to make the equations more convenient for numerical computations, the radial independent variable r is replaced by the new variable $y = r^2/2$. There are three dependent variables: the streamfunction $\Psi(z,y)$, the circulation function $C(z,y) \equiv rV_\theta$, and the so-called blade wrap angle $f(z,y)$ which describe the shape of a S_2-surface.

If the real blades thickness is to be taken into account, the dimensionless blade blockage coefficient $b(z,y)$ must be know within the bladed regions. Moreover, the function $\mathrm{d}E/\mathrm{d}\Psi$ (for absolute flow), or $\mathrm{d}E_R/\mathrm{d}\Psi$ (for relative flow) must be known as function of Ψ from flow configuration upstream the computational domain.

The governing equations for axisymmetric turbomachinery swirling flow are

$$\frac{1}{2y}\frac{\partial}{\partial z}\left(\frac{1}{b}\frac{\partial \Psi}{\partial z}\right)+\frac{\partial}{\partial y}\left(\frac{1}{b}\frac{\partial \Psi}{\partial y}\right)-b\frac{\partial E_R}{\partial \Psi}=\frac{\partial f}{\partial y}\frac{\partial C}{\partial z}-\frac{\partial f}{\partial z}\frac{\partial C}{\partial y} \quad (\mathrm{a})$$

$$\frac{1}{b}\left(\frac{\partial \Psi}{\partial y}\frac{\partial f}{\partial z}-\frac{\partial \Psi}{\partial z}\frac{\partial f}{\partial y}\right)=\frac{C}{2y}-\Omega \quad (\mathrm{b}) \qquad (22)$$

where $f(z,y)$ corresponds to a streamsurface for relative flow, and Ω is the runner angular speed. By setting $\Omega = 0$ we obtain the corresponding equations for absolute flow.

Since we have only two equations and three unknown functions, either $C(z,y)$ or $f(z,y)$ must be specified, while $\Psi(z,y)$ is allways computed. As a result we have two alternatives for using Eqs.(22):

- Analysis mode: for a given streamsurface (blade) shape, $f(z,y)$, compute the corresponding axisymmetric velocity field; the velocity components in a meridian half-plane are obtained from the streamfunction $\Psi(z,y)$, while the circumferential velocity component is given by the circulation function $C(z,y)$;
- Design mode: for a given distribution fo the circulation function $C(z,y)$, compute the shape of S_2-streamsurfaces, $f(z,y)$, and the streamfunction $\Psi(z,y)$; a stacking curve from hub to shroud should be given on the inlet section as initial condition to integrate (22)(b). Actual blade sections are further

designed on axisymmetric S_1-streamsurfaces obtained by revolving $\Psi = \text{constant}$ curves about the symmetry axis.

Efficient numerical approaches for solving directly the three-dimensional inviscid or viscous flow are now readily available, thus making the above "analysis mode" obsolete. However, the design mode is still the first choice for hydraulic turbines and pumps preliminary design.

An iterative algorithm for solving Eqs.(22) starts with solving the homogeneous version of (22)(a) and obtain a first approximation for $\Psi(z,y)$. In design mode, this approximation together with the prescribed circulation function $C(z,y)$ is used to integrate (22)(b) and obtain an approximation for $f(z,y)$. The right-hand side in (22)(a) can now be evaluated, and the next iteration can be started by computing a new approximation for $\Psi(z,y)$.

Swirl Generator Blade Design

Our preliminary swirling flow analysis and design [8] let us to the velocity profiles required downstream the guide vanes and runner blades, respectively, as shown in **Fig. 3**. The blading region corresponds to an annular space with hub radius of 45 mm and casing radius of 75 mm. The overall water discharge supplied by our test rig is 30 liter/second.

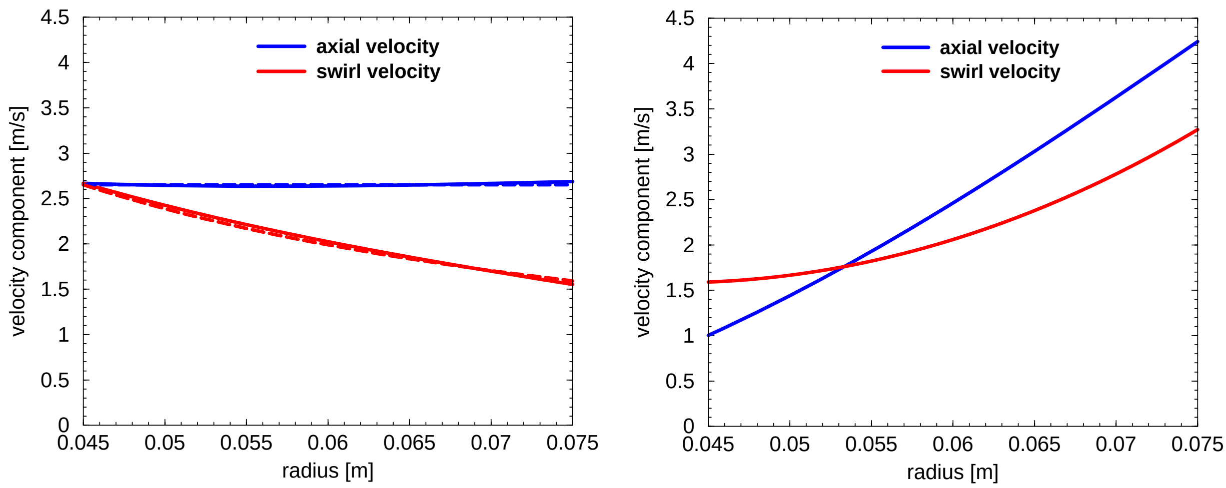

Fig. 3: Axial and circumferential velocity profiles downstream the guide vanes with $\alpha^{(1)}_{\text{hub}} = 45^\circ$, $\alpha^{(1)}_{\text{tip}} = 60^\circ$, and downstream the free runner with $\beta^{(2)}_{\text{hub}} = 25^\circ$, $\beta^{(2)}_{\text{tip}} = 55^\circ$.

Given these preliminary design data, we choose both guide vanes and runner blades axial extent of 60 mm. The inlet flow for guide vanes correspond to a uniform axial flow without swirl. Downstream the guide vanes we prescribe the swirl velocity from **Fig. 3** left, resulting in the dimensionless blade loading distribution shown in **Fig. 4**.

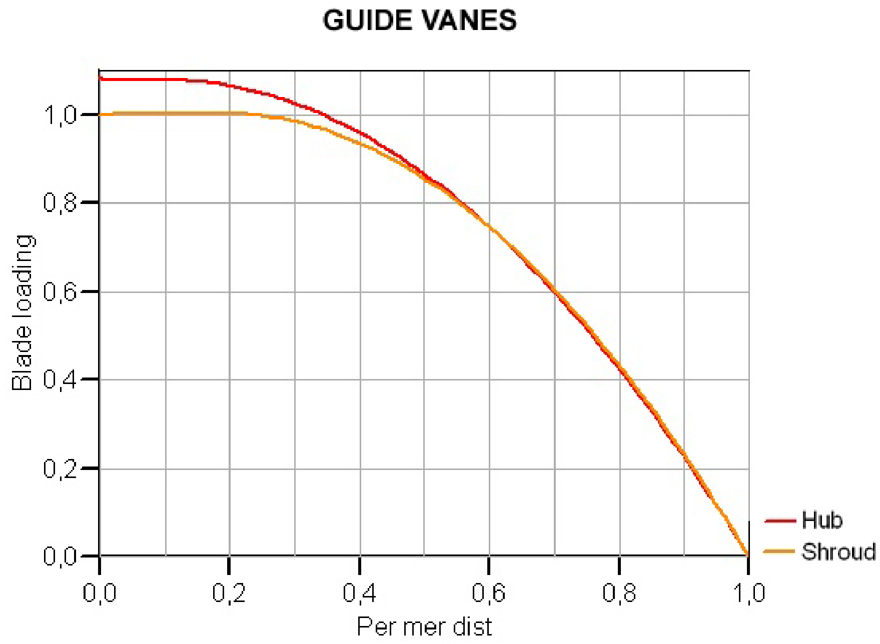

Fig. 4: Guide vanes loading from leading edge to trailing edge.

For the guide vanes, the circulation function $C(z,y) \equiv rV_\theta$ is made dimensionless with respect to the discharge velocity 2.653 m/s, and the average radius of 60 mm. Actually, we plot in **Fig. 4** the quantity $\partial(rV_\theta)/\partial z$ at the hub and shroud, respectively. We consider 13 guide vanes, and the result is shown in **Fig. 5**.

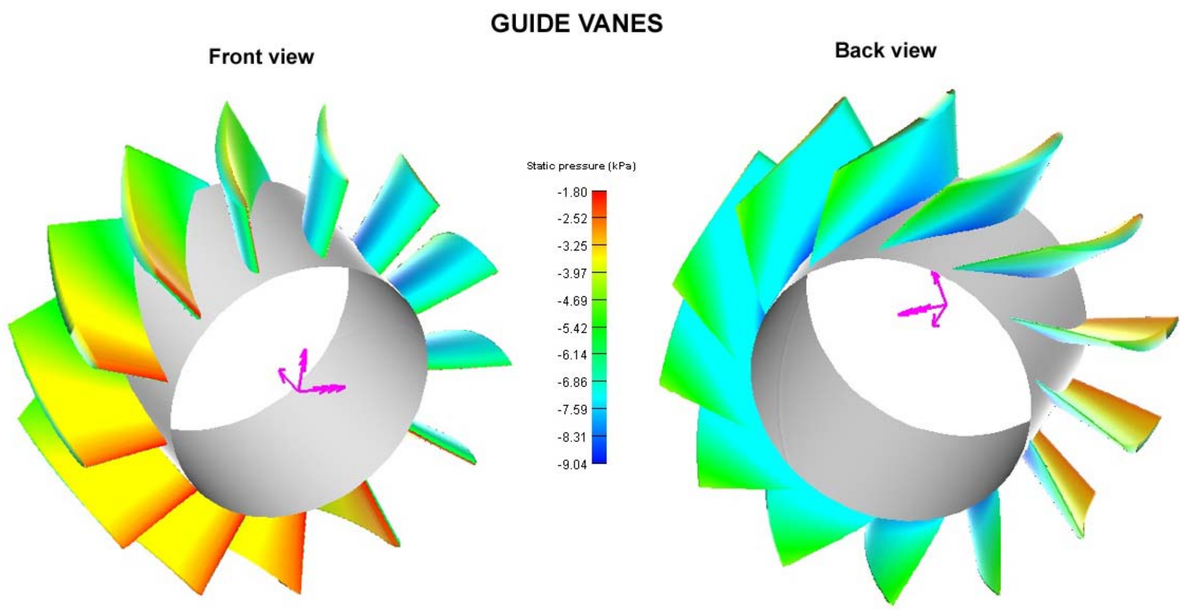

Fig. 5: Guide vanes shape and pressure distribution.

The pressure distribution on three cylindrical cross-sections of the guide vane is shown in **Fig. 6**.

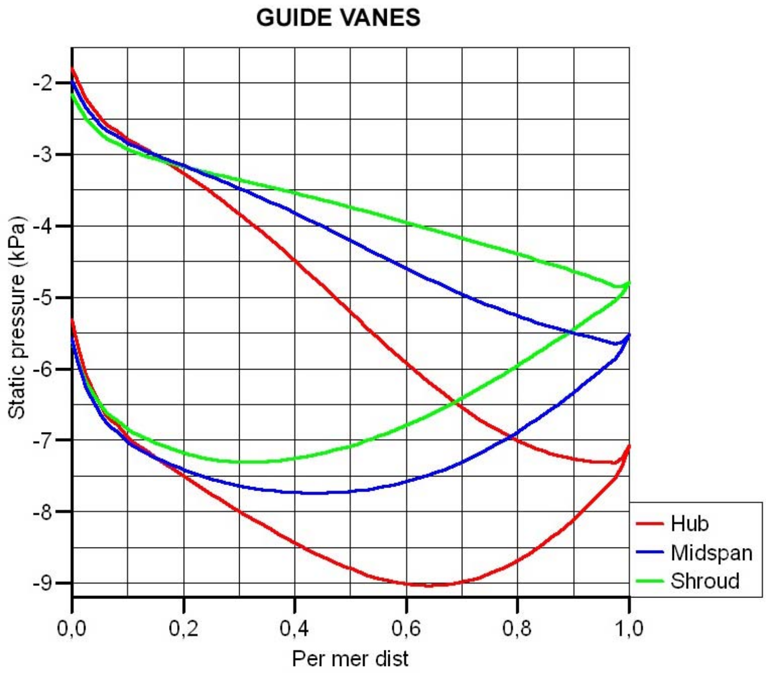

Fig. 6: Pressure distribution on three sections of guide vanes.

For the runner we choose 8 blades, and the angular speed has been determined in [8] such that the overall torque vanishes, $\Omega = 83.24\ rad/s$, i.e. 795 rpm. The reference velocity will be 4.99 m/s, corresponding to the transport velocity at the reference radius of 60 mm.

Fig. 7: Runner blades loading from leading edge to trailing edge.

The runner blades loading shown in **Fig. 7** reveals that near the hub the runner behaves like a turbine, while near the tip it behaves like a pump. As a result, a deficit of total pressure and velocity is generated near the hub, with a corresponding excess near the blade tip.

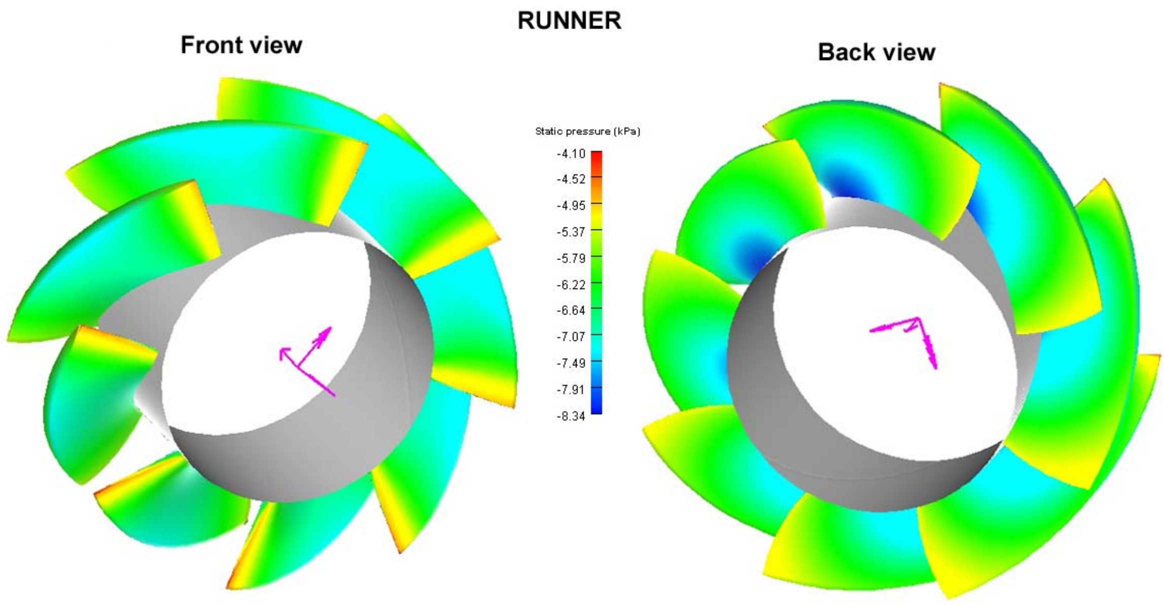

Fig. 8: Runner blades shape and pressure distribution.

The result for runner blades design is shown in **Fig. 8**, with the pressure distribution on three cylindrical section along the blade span plotted in **Fig. 9**.

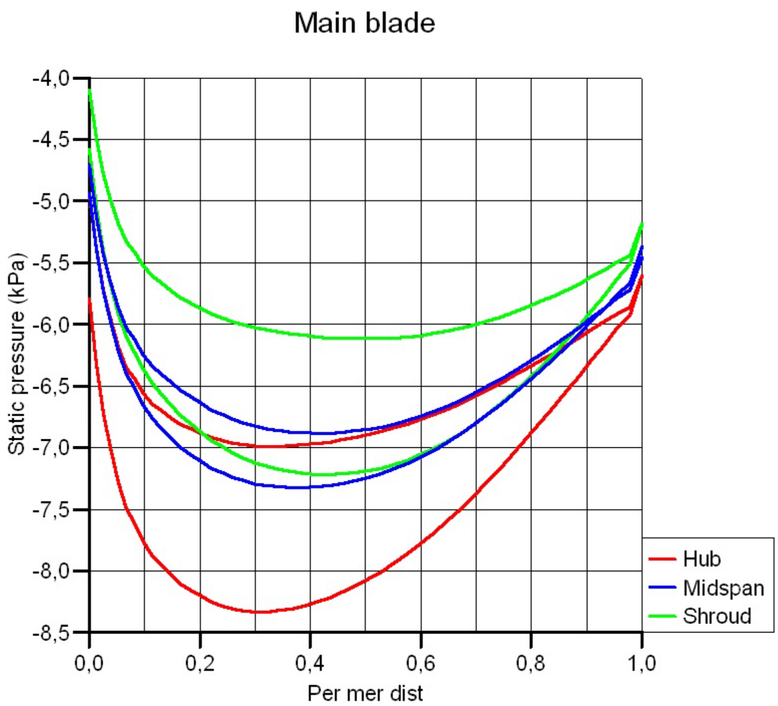

Fig. 9: Pressure distribution on three sections of runner blades.

Conclusion

The paper presents the theoretical developments for quasi-three-dimensional inverse design of turbomachinery blades, together with an application for a swirling flow generator. Both stationary guide vanes and rotating runner blades are designed such that a prescribed absolute and relative flow deflection is achieved in the bladed region. The resulting 3D blade shape is directly used for manufacturing using the rapid prototyping technology.

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