THE STABILITY OF THE SWIRLING FLOWS WITH APPLICATIONS TO HYDRAULIC TURBINES

Forica Ioana DRAGOMIRESCU
Department of Mathematics, "Politehnica" University of Timisoara, Romania

Diana Alina BISTRIAN
Engineering Faculty of Hunedoara,"Politehnica" University of Timisoara, Romania

Sebastian MUNTEAN
Centre of Advanced Research in Engineering Sciences, Romanian Academy - Timisoara Branch, Romania

Romeo SUSAN-RESIGA
Department of Hydraulic Machinery,"Politehnica" University of Timisoara, Romania

Abstract

A Chebyshev tau spectral method for the investigation of the eigenvalue problem governing the linear stability of swirling flows is presented. The accuracy of the developed algorithm imposed by the complicated boundary conditions corresponding to the real flow case of a fluid downstream a Francis turbine runner was validates on a benckmark model and the results proved to agree quite well. The results for the case of the hydraulic turbine runner are compared in this preliminary investigation with the existing ones in the case of steady axisymmetric swirling flow [16] and good agreements were found.

Keywords: swirling flow, Batchelor vortex, eigenvalue problem, tau - spectral method.

1. INTRODUCTION

The presence of a large variety of vortex flows in nature and technology has raised many theoretical and numerical problems concerning the stability of such structures. The numerical simulation is the main instrument to investigate this type of three dimensional unsteady flows. However, the simulation requirements are very expensive even with very powerful computer resources. In these conditions, stability analyses of vortex motions that can help to better understand the dynamical behavior of the flow by offering a significant insight for the physical mechanics of the observed dynamics become very important in flow control problems.

Most of the vortex stability analyses concerned axisymmetrical vortices with axial flow [8] in order to explain the vortex breakdown phenomenon observed experimentally for the first time on delta wings [13], in pipes [14] and in cylinders with rotating ends [7]. Obviously, the axial symmetry hypothesis is a major simplification having the main benefit of dramatically reducing the computational cost [15]. On the other hand, it introduces important

* *Corresponding author:Dragomirescu, F.I.*, Department of Mathematics,University „Politehnica" of Timisoara, Piata Victoriei, No.2, 300006, Timisoara, Romania, phone: +40-0256 403099, email: ioana.dragomirescu@mat.upt.ro

limitations as far as the three-dimensionality and unsteadiness of the flow are concerned. Essentially, an axisymmetric flow solver provides a circumferentially averaged velocity and pressure fields that is why it is used as a basic flow for linear stability analysis.

One of the reasons to deal with a mathematical model governing the linear stability of the mechanical equilibria of a swirling flow is to investigate the evolution of the amplitudes of the velocity and the pressure perturbations fields, respectively.

In most cases, the spatially or temporal stability (classified for open flows as in [8]) under infinitesimal perturbations is reduced to the study of an algebraic eigenvalue problem which imply solving a dispersion relation connecting in fact the frequency ω and the axial wavenumber k as a consequence of the condition that nontrivial eigenfunctions exist. The instability of the flow is described by the dispersion relation in the spectral space (k,ω) corresponding to the spatio-temporal evolution of the most unstable mode in the physique space (x,t). Most of the investigations [9], [12] concerned the values of the nondimenional parameters for which the vortex become unstable in the case of either a spatial stability or temporal stability analysis.When the complex frequency $\omega = \omega_r + i\cdot\omega_i$, $\omega_r = \mathrm{Re}(\omega)$, $\omega_i = \mathrm{Im}(\omega)$ is determined as a function of the real wave number k a temporal stability analysis is in fact perfomed. Conversely, solving the dispersion relation for the complex wave number, $k = k_r + i\cdot k_i$, $k_r = \mathrm{Re}(k)$, $k_i = \mathrm{Im}(k)$, when ω is given real leads to the spatial branches $k(\omega,R)$ where by R we denoted the set of all other physical parameters involved. In both cases, the sign of the imaginary part indicates the decay or either the growth of the disturbance.

Although a spatial stability analysis implies the investigation of a nonlinear eigenvalue problem, this type of analysis directly provides the frequency ranges of the most unstable modes. More than that, the spatial stability results can be directly compared to the experimental ones since usually, in experiments, an excitation is applied to a point in the flow and then, the effect of the excitation is studied as the flow evolves downstream.

In a spatial stability analysis, useful conclusions can be drawn considering k^2 as a function of the real frequency ω. Since $k^2 < 0$ implies an imaginary eigenvalue k, for the flow moving downstream the current section the only physically acceptable case is the one for which the exponential factor $e^{-|k|z}$ holds. A detailed classification of the flows with respect to the sign of the k^2 term was given by Benjamin [4].

Linearization of axisymmetric steady flow of incompressible and inviscid fluid using linearized Bragg - Hawthorn equation in order to analysis the stability of swirling flow downstream to the Francis turbine was used by Resiga et al [16]. In this case, the problem for the streamfunction is defined then the eigenvalue as the axial wavenumber k is obtained. Applying to the real axial and circumferential velocity profiles downstream a Francis runner at different operating points the swirling flow stability is evaluated. Following Benjamin's theory of finite transitions between frictionless cylindrical flows, an eigenvalue analysis of the linearized problem was performed. It was shown that the swirl reaches a critical state at discharge $\varphi = 0.365$. For larger discharge the flow ingested by the draft tube is supercritical, while at lower discharge it is subcritical. The critical state occurs quite close to the discharge $\varphi = 0.370$ where a sudden variation in the draft tube pressure recovery, as well as in the overall turbine efficiency, is experimentally observed. For the particular turbine under investigation this discharge value happens to correspond to the best efficiency point, leading to a negative impact on the turbine regulation.

Our method is first validated on a benchmark model of the Batchelor vortex and then applied to a real flow configuration at the outlet of a Francis turbine runner. In a preliminary analysis viscous losses can be considered negligible at the design operating point so the inviscid fluid assumption is taken. The simple stability analysis carried out in Resiga et al [16] can be recovered as a particular case here for $m = 0$ and $\omega = 0$.

The paper is organized as follows. The second section of the paper is splitted in two subsections. The eigenvalue problem governing the linear stability analysis for inviscid swirling flows against normal mode perturbations is defined in the first part of the section and then the proposed mathematical method, i.e. the tau spectral method is presented pointing out its advantages for this case. The numerical results validated using the benchmark model of the Batchelor vortex are presented in the third section of the paper. All results are compared to existing ones [3], [12]. A particular case flow occurring in Francis turbines operating at partial discharge is examined in Section 4 using the proposed method. The main results of the paper are summarized in Section 5.

2. MATHEMATICAL FORMULATION

2.1 Governing equations and boundary conditions

The most convenient choice for the formulation of the Euler equations for the inviscid flow and the continuity equation is the cylindrical coordinates system

$$\frac{\partial \underline{V}}{\partial t} + (\underline{V} \cdot \nabla)\underline{V} = -\frac{1}{\rho}\nabla p, \; \nabla \cdot \underline{V} = 0, \tag{1}$$

with the velocity field $\underline{V}$ and the pressure p depending on the axial coordinate z, the radial coordinate r and the azimuthal component θ, respectively.

In order to derive the disturbance equations, the velocity and the pressure perturbation fields are divided into their mean and perturbation parts, $V = \underline{V} + \underline{v}$, $P = p + \pi$ with the disturbances assumed small. Since the base flow obey the Euler equations (1) the evolution of such small perturbations of the basic flow is governed by the following dimensionless linearized equations

$$\nabla \cdot \underline{v} = 0, \quad \frac{\partial \underline{v}}{\partial t} + (\underline{V} \cdot \nabla)\underline{v} + (\underline{v} \cdot \nabla)\underline{V} = -\frac{1}{\rho}\nabla\pi \tag{2}$$

called **linearized Euler equations**. In the linearization process the second order terms in the small perturbations were neglected.

Assuming a steady columnar flow the velocity profile is written:

$$\underline{V}(r) = [U(r), 0, W(r)] \tag{3}$$

where U and W represent the axial and the azimuthal velocity, respectively, both depending only on the radial coordinate r. Next, we consider the normal mode form of the small perturbations [12]

$$[\underline{v}(t,z,r,\theta), \pi(t,z,r,\theta)] = [F(r), iG(r), H(r), P(r)]\, e^{i(kz + m\theta - \omega t)}, \tag{4}$$

where F, G, H, P represent the complex amplitudes of the perturbations, k is the axial wave number complex, m is the azimuthal wave number entire and ω represents the complex frequency. The factorization with respect to the axial coordinate z is allowed by the assumption on an axisymmetric parallel flow, the factorization in the azimuthal direction can be considered based on the axisymmetric flow assumption also. A linear stability study implies infnitesimal type perturbations so a factorization in time can be considered.

Introducing the factorization form (4) into the linearized Euler equations (2) we obtain the following set of first order differential equations with variable coefficients leading to singularities from [12]:

$$k\, r\, F + G + r\, G' + m\, H = 0, \tag{6a}$$

$$kUG - \omega G + \frac{mWG}{r} + \frac{2WH}{r} - P' = 0\,, \tag{6b}$$

$$rHkU - rH\omega + m(HW + P) + WG + rGW' = 0\,, \tag{6c}$$

$$FkU - F\omega + \frac{FmW}{r} + U'G + kP = 0\,, \tag{6d}$$

where prime denotes differentiation with respect to the radius. This homogenous first order differential system is completed with the boundary conditions from Batchelor and Gill [3] given in detail also in [9]. The boundary conditions at the axis have the form

$$(|m| > 1), \quad F = G = H = P = 0\,, \tag{7a}$$

$$(m = 0), \quad G = H = 0, F, P \text{ finite,} \tag{7b}$$

$$(m = \pm 1), \quad H \pm G = 0, F = P = 0. \tag{7c}$$

At the outer wall all components of the velocity are enforced to vanish, this condition leads to corresponding boundary conditions for $r = r_{\max}$. The pressure field must balance the viscous forces. We get

$$F, G, H, P \to 0, r = r_{\max}\,, \tag{7d}$$

$r_{\max} = 3$ [12], chosen also by Lessen and Paillet [10] to start their asymptotic solution.

For the case of the flow downstream a Francis turbine runner, the physical condition that the radial amplitude of the velocity perturbation at the wall is neglijable, i.e. $G(r_{\max}) = 0$, is valid. In these conditions, at $r = r_{\max}$ the outer boundary conditions can be deduced from the system Eq.(6a)-Eq.(6d). They have the form

$$(m = 0), \quad G = 0,\ H = 0,\ P' = 0, kF + G' = 0, F(kU_{r\max} - \omega) + kP = 0\,, \tag{7e}$$

$$(m \neq 0), \quad \frac{2W_{r\max}H}{r_{\max}} - P' = 0,\ G = 0,\ r_{\max}H\left(kU_{r\max} - \omega + \frac{mW_{r_{\max}}}{r_{\max}}\right) + mP = 0,$$
$$F\left(kU_{r\max} - \omega + \frac{mW_{r_{\max}}}{r_{\max}}\right) + kP = 0, \tag{7f}$$

where $U_{r_{\max}}$ and $W_{r_{\max}}$ represents the axial and the azimuthal velocity of the basic flow evaluated at the wall, i.e. $r = r_{\max}$. In Eq.(7e)-Eq.(7f) the dimensionless with respect to runner outlet radius $r_{\max}$ takes the value 1.05 as in [16]. The system Eq. (6a) –Eq.(6d), Eq. (7e)- Eq. (7f) represents the eigenvalue problem governing the linear stability of the mechanical equilibria of the fluid in our real flow problem in a Francis turbine runner.

2.2 A shifted Chebyshev polynomials based tau method

In [5] a numerical procedure to investigate the system Eq. (6a)-Eq. (6d) for the case $|m| > 1$, i.e. Dirichlet boundary conditions, was developed. The key issue of the proposed pseudo-spectral collocation method, one of the most used technique for numerical investigations in hydrodynamic stability problems, was the choice of the modal trial basis functions. It concerned orthogonal expansions functions satisfying the boundary conditions Eq. (7a). The selection of the spaces involved in the discretization process was motivated by the need to adapt the grid points to the singularities of the underlying solution. Moreover, the Galerkin method, also used for an analytical and numerical investigation of the eigenvalue problem in [5] is far to difficult to apply for nonperiodic boundary conditions.

The sophisticated boundary conditions Eq.(7e), Eq.(7f) corresponding to the real flow case in a Francis tubine runner motivated the use of the Chebyshev tau method suitable for non-periodic problems with complicated boundary conditions. There are two possible approaches of the system at this point. The first one imply a transformation of the physical domain onto the standard interval of the definition of the Chebyshev polynomials. A linear transformation of the form $r=\frac{r_{\max}}{2}(x+1)$ can be used to map the interval $[0,r_{\max}]$ on the interval $[-1,1]$. For the second approach, instead of using classical Chebyshev polynomials, one can use shifted Chebyshev polynomials directly defined on the physical interval of the problem. This choice is motivated by the form of the nonconstant coefficient of the unknown functions from (6a) - (6d) and also by the orthogonality of the shifted class directly in the physical space and therefore there will be no need for a numerical interpolation of the jacobian.

Let us define the perturbation amplitudes as a fnite series of Chebyshev polynomials

$$(F,G,H,P)=\sum_{k=1}^{N}(f_k,g_k,h_k,p_k)\cdot T_k^* \tag{8}$$

where T_k^* are shifted Chebyshev polynomials on the physical domain $[0,r_{max}]$. In order to reduce the system Eq.(6a)-Eq.(6d) to a finite dimensional algebraic system in the expansion coefficients only, we impose the condition that each equation of the system to be orthogonal on T_i^*, $i=0,...,N-2$, in the Hilbert space $L_w^2(0,r_{\max})$, with $w(r)=\frac{r_{\max}}{2\sqrt{r(r_{\max}-r)}}$.

Introducing the notation

$$I_{ijklm}^U=\left(r^k(U^l)^{(m)}T_i^*,T_j^*\right)_w,\ I_{ijklm}^W=\left(r^k(W^l)^{(m)}T_i^*,T_j^*\right)_w, \tag{9}$$

with m the derivation order, the first truncated $4(N-2)$ equations are

$$k\sum_{j=1}^{N}\left(f_jI_{ij100}^U\right)+g_ir_{max}c+g_2\frac{2}{r_{max}}I_{i1100}^U+\sum_{\substack{j=3\\ j\ odd}}^{N}g_j\frac{2(j-1)}{r_{max}}\left[\sum_{\substack{r=j-1\\ j\ even}}^{2}2I_{ir100}^U\right]+$$

$$+\sum_{\substack{j=4\\ j\ even}}^{N}g_k\frac{2(j-1)}{r_{max}}\left[\sum_{\substack{r=j-1\\ j\ odd}}^{2}\left(2I_{ir100}^U\right)+I_{i1100}^U\right]+mh_ir_{max}c=0 \tag{10a}$$

$$k\sum_{j=1}^{N}g_jI_{ij010}^U-\omega g_ir_{max}c+m\sum_{j=1}^{N}g_jI_{ij-110}^W+2\sum_{j=1}^{N}h_jI_{ij-110}^W-p_2\frac{2}{r_{max}}A_{i1}-\sum_{\substack{j=3\\ j\ odd}}^{N}p_j\frac{2(j-1)}{r_{max}}\left[\sum_{\substack{r=j-1\\ j\ even}}^{2}2A_{ir}\right]-$$

$$-\sum_{\substack{j=4\\ j\ even}}^{N}p_j\frac{2(j-1)}{r_{max}}\left[\sum_{\substack{r=j-1\\ j\ odd}}^{2}\left(2A_{ir}\right)+A_{i1}\right]=0 \tag{10b}$$

$$k\sum_{j=1}^{N}h_jI_{ij110}^U-\omega\sum_{j=1}^{N}h_jI_{ij100}^U+m\sum_{j=1}^{N}h_jI_{ij010}^W+m\,p_ir_{max}c+\sum_{j=1}^{N}g_j\left(I_{ij010}^W+I_{ij111}^W\right)=0 \tag{10c}$$

$$k\left(\sum_{j=1}^{N} f_j I_{ij010}^{U} + p_i r_{max} c\right) - \omega f_i r_{max} c + m \sum_{j=1}^{N} f_j I_{ij-110}^{W} + \sum_{j=1}^{N} g_j I_{ij011}^{U} = 0$$

(10d)

for $i = 1..N-2$, where the number c being defined as $c = \begin{cases} \pi/2, & i = 1 \\ \pi/4, & i = 2..N-2 \end{cases}$ and $A \in M_{N-2}$ is square $(N-2)\times(N-2)$ matrix, with $A_{11} = r_{max}\pi/2$, $A_{mm} = r_{max}\pi/4$, $m = 2..N-2$, $A_{mn} = 0$, $m \neq n$. The eight remaining equations are provided by the boundary conditions. The eigenvalue problem is obtained as a system of $4N$ equations written in a matriceal form as

$$kM_k \bar{s} = M\bar{s}, \quad \bar{s} = \left(\bar{f}, \bar{g}, \bar{h}, \bar{p}\right) \tag{11}$$

with $\bar{*} = (*_1, ..., *_N)$, $* \equiv f, g, h, p$.

3. The Batchelor vortex case

The basic flow under consideration for a validation of the proposed method is the Batchelor vortex case or the q-vortex [12] characterized by the velocity field

$$U(r) = a + e^{-r^2}, \quad W(r) = \frac{q}{r}\left(1 - e^{-r^2}\right) \tag{12}$$

with q representing the swirl number defined as the angular momentum flux divided by the axial momentum flux times the equivalent nozzle radius and a provides a measure of free-stream axial velocity. The model Eq. (6a) - Eq. (6d) with Dirichlet boundary conditions to infinity has been used by Olendraru et al [12] to study the inviscid instability of this type of vortex using a shooting method. The properties of the Batchelor vortex are pointed out by considering them as functions of the swirl ratio q and the external flow parameter a.

a)	Eigenvalue with largest imaginary part $k_{cr} = (k_r, k_i)$	*Relative error*
Collocation boundary adapted results[5]	(0.50842,–0.14243)	0.6%
Tau method results	(0.46375,- 0.27935)	3%

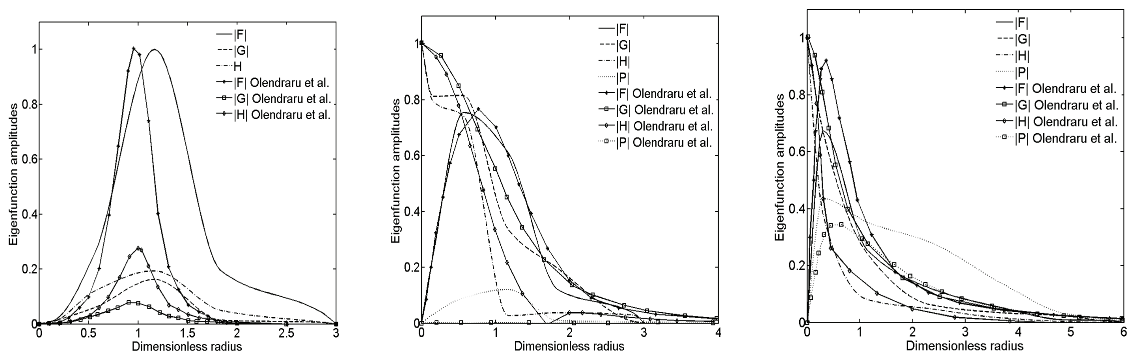
b)	Critical distance of the most amplified perturbation r_c	*Relative error*
Collocation boundary adapted results[5]	0.959	4%
Tau method results	1.166	10%

***Tab. 1**. Comparative results of the most amplified downstream k-spatial wave at* $a = 0$, $q = 0.1$, $\omega = 0.01$ *for the case of the counter-rotating mode* $m = -3$*: a) the eigenvalue with the largest imaginary part; b) the critical distance of the most amplified perturbation.*

In the particular case $a = 0$, Lessen and Paillet [10] investigated stability characteristic of the velocity profile defined by Eq. (12). The solution was started with a Frobenius series at $r = 0$ and a Taylor series expansion was used to integrate from both limits and the condition that the solution matched at some intermediate points was imposed. It was proven that for $q \geq 1.5$ all unstable modes are highly damped and stabilized.

In order to compare our results with the ones from [12] numerical evaluations of the axial wavenumber k and the critical distance were obtained for various values of the spectral parameter *N*. In Table 1 we present these values in comparison with the ones from Olendradru [12] and the ones obtained by us using a shifted Chebyshev polynomials based collocation method. The numerical results obtained in [5] are better, however, the tau method allowed us to investigate the cases where the condition $|m| > 1$ is not valid also.

The eigenfunctions F, G, H, P corresponding to the largest eigenvalue for subcritical flows are obtained. All these results provide information upon the fluid flow for given parameters, assessing the vortex breakdown phenomena (Fig.1).



a) b) c)

Fig. 1. Absolute values of eigenfunction amplitudes computed a) $a = 0$, $q = 0.1$, $\omega = -0.01$, *for mode* $m = -3$ *and* $k = (0.46, -0.27)$, *for* $N = 7$ *Chebyshev expansion terms; b)* $m = 1$, $a = 0$, $q = 0.7$, $\omega = 0.0425$, $k = 0.49$ *using* $N = 7$ *expansion terms; c)* $m = -1$, $a = -1.268$, $q = 0.6$, $\omega = -0.78$, $k = (0.57, -1.358)$ *using* $N = 8$ *expansion terms.*

4. Application to a fluid flow downstream a Francis turbine runner

A particular case arises in the Francis turbines operating at partial discharge. The swirling flow downstream the runner becomes unstable inside the draft tube cone, with the development of a precessing helical vortex (also called vortex rope) and associated severe pressure fluctuations. A tridimensional simulation to accurately simulate the vortex rope requires a large amount of computational resources first of all, due to the fact that the vortex rope often covers most of the draft tube. Moreover, accurate predictions on this unsteady phenomenon imply that the flow problem is solved many times at very small time steps.

In Resiga et al. [16] an analytical representation for the axial and circumferential velocity components measured on survey section was developed. It was found that the mean swirling flow downstream the Francis turbine runner can be accurately represented as a superposition of three distinct vortices

$$U(r) = U_0 + \underbrace{U_1\, exp\left(-\frac{r^2}{R_1^2}\right)}_{co-flowing} + \underbrace{U_2\, exp\left(-\frac{r^2}{R_2^2}\right)}_{counter-flowing} \tag{15a}$$

$$W(r) = \Omega_0 r + \underbrace{\Omega_1 \frac{R_1^2}{r}\left[1 - exp\left(-\frac{r^2}{R_1^2}\right)\right]}_{counter-rotating} + \underbrace{\Omega_2 \frac{R_2^2}{r}\left[1 - exp\left(-\frac{r^2}{R_2^2}\right)\right]}_{co-rotating} \tag{15b}$$

The simple stability analysis carried out in Resiga et al [16] can be recovered as particular case for $m=0$ and $\omega=0$. In these conditions, the system Eq. (6a) –Eq.(6d), Eq. (7e)- Eq. (7f) reduces to a much simpler form in which the number of components of the eigenvector was reduced

$$\begin{aligned} &krF + G + rG' = 0, \\ &kUG - P' = 0, \\ &kUf + U'G + kP = 0 \end{aligned} \tag{16}$$

with the corresponding boundary conditions

$$G = F' = P' = 0 \text{ at } r = 0, r_{\max} . \tag{17}$$

Obviously, a simple handling of the equations from Eq. (16) should lead to an eigenvalue problem written in one perturbation function only equivalent to the one from [16] written in the perturbation of the streamfunction of the basic flow ψ when the auxiliary variables are used. The amplitude of the radial velocity perturbation $|G|$ should then be proportional to $\frac{1}{r}\psi$ and the amplitude of the axial velocity perturbation with $\frac{1}{r}\frac{d\psi}{dr}$.

As a result, Figure 2 presents the radial perturbation computed using the algorithm presented in this paper against the values obtained in Resiga et al. [16]. One can observe a shifted distribution of the radial eigenfunction |G| computed based on our algorithm but additional procedure must be considered in order to increase the accuracy. Due to the linearization procedure, the perturbations are disconnected by the basic flow. Consequently, the eigenfunction values are not significant but the forms provide the relevant information.

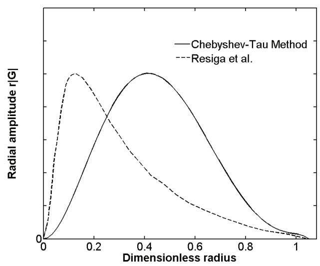


Fig. 2. Eigenfunction $|G|$ for the eigenvalue with the largest (negative) imaginary part in the case $m=0, \omega=0$.

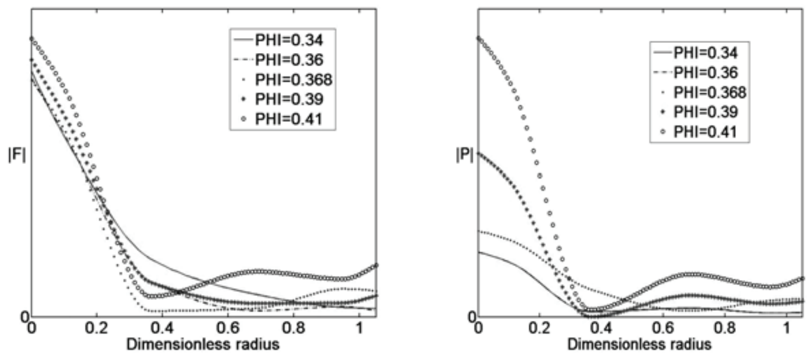


Fig. 3: Eigenfunctions axial |F| and pressure |P| at $m=0$ versus dimensionless radius. The eigenfunctions are computed for velocity profiles downstream to the Francis turbine using data from Resiga et al [16] at different discharge coefficients (φ=0.34, 0.36, 0.368,0.39, 0.41).

The eigenfunctions |F| axial and |P| pressure at m=0 in terms of dimensionless radius are presented in Fig. 3 for velocity profiles downstream to the Francis runner at different discharge coefficients. The axial eigenfunction of velocity |F| is most unstable to axi-symmetric perturbation m=0, near the axis at all values of the discharge coefficients investigated due to the crown weak [17. Also, the most unstable eigenfunction of pressure |P| at axi-symmetric perturbation m=0 is near to the axis at largest discharge φ=0.41. The sensitivity at axi-symmetric perturbation m=0 near to the axis decreases as long as the discharge goes down. A preliminary conclusion still under study is that the radial eigenfunction of the velocity, i.e. |G| seems to be sensitive to axi-symmetric perturbation m=0 at largest discharge.

4. CONCLUSIONS

In this paper a polynomials based tau-numerical procedure to investigate the spatial stability of a swirling flow subject to infinitesimal perturbations was developed. Using a shifted Chebyshev approach, our numerical procedure directly provided relevant information on perturbation amplitude for stable or unstable induced modes, the maximum amplitude of the most unstable mode and the critical distance where the perturbation is the most amplified.

It is clear that the method is not the most precise one. However, the major advantage is that it allows a good handling of the complicated boundary conditions. Another important aspect that must be pointed out is that the numerical approximations of the unknown perturbation fields are reached directly in the physical space due to a careful selection of the discretization spaces. A preliminary conclusion can be drawn: the non-symmetric boundary conditions have a major influence on the stability domain.

The choice of the method was assessed underlying the necessity to implement an eigenvalue problem with sophisticated boundary conditions, governing the stability of the hydrodynamic system. The results obtained were validated for the axi-symmetric case $m = 0$ where good agreements were found. A complete analysis of the case $m = 0$ with an improved accuracy is in order. Further, the cases where helical vortex breakdown occurs, i.e. $m = \pm 1$ will be investigated.

5. ACKNOWLEDGEMENTS

The present work has been supported by Romanian Academy program “Hydrodynamics Optimization and Flow Control of the Hydraulic Turbomachinery in order to Improve the Energetic and Cavitational Performances”.

6. REFERENCES

[1] Alekseenko, S.V., Kuibin, P.A., Okulov, V.L.: *Theory of concentrated vortices*, Springer-Verlag Berlin Heidelberg, 2007.

[2] Avellan, F.: Flow Investigation in a Francis Draft Tube: The FLINDT Project, *Proc. 20th IAHR Symposium on Hydraulic Machinery and Systems*, Charlotte, USA, 2000.

[3] Batchelor, G.K, Gill, A.E.: Analysis of the stability of axisymmetric jets, *J. Fluid Mech.,* Vol. 14, 1962, pp. 529 -551.

[4] Benjamin, T.J.: Theory of the vortex breakdown phenomenon, *J. Fluid Mech*., **14,** 1962, pp. 593-629.

[5] Bistrian, D., Dragomirescu, F. I., Muntean, S., Topor, M.: Numerical methods for convective hydrodynamic stability of swirling flows, *13th WSEAS Conference on Systems*, 22-25 July, Rodos, 2009, pp. 283-288.

[6] Bourne D.: Hydrodynamic stability, the Chebyshev tau method and spurious eigenvalues. *Continuum Mech. Thermodyn.*,Vol. 1**5**, Springer-Verlag, 2003, pp. 571-579.

[7] Escudier, M.: Observations of the flow produced in a cylindrical container by a rotating endwall, *Experiments in Fluids*, Vol. 2, 1984, pp. 189- 196.

[8] Huerre P., Monkewitz, P.A.: Local and global instabilities in spatially developing flows, *Annu. Rev. Fluid Mech.*, Vol. 22, 1990, pp. 473-537.

[9] Khorrami M. R., Malik M.R., Ash R.L.: Application of spectral collocation techniques to the stability of swirling flows, *J. Comput. Phys.*, Vol. 81, 1989, pp. 206–229.

[10] Lessen M., Paillet F.: The stability of a trailing line vortex. Part 2. Viscous theory, *J. Fluid Mech.*, 65**,** 1974, pp. 769-779.

[11] Loiseleux T., Chomaz J.M., Huerre P.: The effect of swirl on jets and wakes : Linear instability of the Rankine vortex with axial flow, *Physiscs of Fluids*, (10)5, 1998, pp. 1120-1126.

[12] Olendraru C., Sellier A., Rossi M., Huerre P.: Inviscid instability of the Batchelor vortex: Absolute-convective transition and spatial branches, *Physics of Fluids*, Vol. (11) 7, 1999, pp. 1805-1820.

[13] Peckham, D.H., Atkinson, S.A.: Preliminary results of low speed wind tunnel tests on a gothic wing of aspect ratio 1.0, *Aero. British Res. Council*, 1957, CP 508, pp. 16-17.

[14] Sarpkaya, T.: On stationary and travelling vortex breakdown, *J. of Fluid Mech.*, Vol.45, 1971, pp.545-559.

[15] Susan-Resiga R., Muntean S., Stein P., Avellan F.: Axisymmetric Swirling Flow Simulation of the Draft Tube Vortex in Francis Turbines at Partial Discharge, *Proc. of the 24th IAHR Symp. on Hydraulic Machinery and Systems, October 27-31, Foz do Iguassu, Brasil,* 2008, Paper No. 13.

[16] Susan-Resiga R., Ciocan G.D., Anton I., Avellan F.: Analysis of the Swirling Flow Downstream a Francis Turbine Runner, *Journal of Fluids Engineering*, Vol. 128 (1), 2006, pp. 177-189.

[17] Susan-Resiga R., Avellan F., Ciocan G.D., Muntean S., Anton I.: Mathematical and numerical modelling of swirling flow in Francis turbine draft tube cone, *Scientific Bulletin of the Politehnica University of Timisoara.* 2005.